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Latent Gaussian Processes for Distribution Estimation of Multivariate Categorical Data

Yarin Gal • Yutian Chen • Zoubin Ghahramani

yg279@cam.ac.uk

Multivariate Categorical Data

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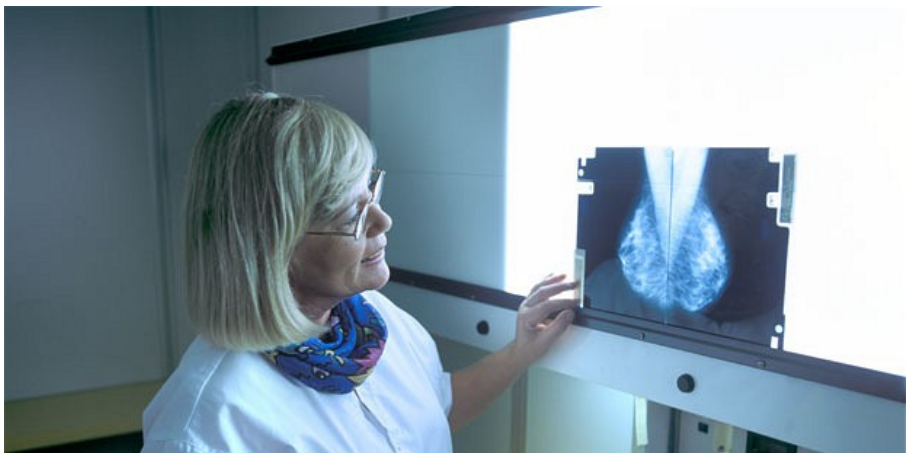

Distribution Estimation

Given

$$D = \{\mathbf{y}_n = (y_{n1}, ..., y_{nD}) \mid n = 1, ..., N\}$$

with

$$y_{nd} \in \{1, ..., K\},$$

we want to **learn a distribution**

$$P(\mathbf{y})$$

from D.

Example: Wisconsin Breast Cancer

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Sample code	Thickness	Unif. Cell Size	Unif. Cell Shape	Marginal Adhesion	Epithelial Cell Size	Bare Nuclei	Bland Chromatin	Normal Nucleoli	Mitoses	Class
1000025	5	1	1	1	2	1	3	1	1	Benign
1002945	5	4	4	5	7	10	3	2	1	Benign
1015425	3	1	1	1	2	2	3	1	1	Benign
1016277	6	8	8	1	3	4	3	7	1	Benign
1017023	4	1	1	3	2	1	3	1	1	Benign
1017122	8	10	10	8	7	10	9	7	1	Malignant
1018099	1	1	1	1	2	10	3	1	1	Benign
1018561	2	1	2	1	2	1	3	1	1	Benign
1033078	2	1	1	1	2	1	1	1	5	Benign
1033078	4	2	1	1	2	1	2	1	1	Benign
1035283	1	1	1	1	1	1	3	1	1	Benign
1036172	2	1	1	1	2	1	2	1	1	Benign
1041801	5	3	3	3	2	3	4	4	1	Malignant
1043999	1	1	1	1	2	3	3	1	1	Benign
1044572	8	7	5	10	7	9	5	5	4	Malignant
1047630	7	4	6	4	6	1	4	3	1	Malignant
1048672	4	1	1	1	2	1	2	1	1	Benign
1049815	4	1	1	1	2	1	3	1	1	Benign
1050670	10	7	7	6	4	10	4	1	2	Malignant
1050718	6	1	1	1	2	1	3	1	1	Benign
1054590	7	3	2	10	5	10	5	4	4	Malignant
1054593	10	5	5	3	6	7	7	10	1	Malignant
1056784	3	1	1	1	2	1	2	1	1	Benign
1057013	8	4	5	1	2	?	7	3	1	Malignant
1059552	1	1	1	1	2	1	3	1	1	Benign
1065726	5	2	3	4	2	7	3	6	1	Malignant
1066373	3	2	1	1	1	1	2	1	1	Benign
1066979	5	1	1	1	2	1	2	1	1	Benign
1067444	2	1	1	1	2	1	2	1	1	Benign
1070935	1	1	3	1	2	1	1	1	1	Benign
1070935	3	1	1	1	1	1	2	1	1	Benign
1071760	2	1	1	1	2	1	3	1	1	Benign
1072179	10	7	7	3	8	5	7	4	3	Malignant

683 patients, 2×10^9 possible configurations

Categorical Latent Gaussian Process

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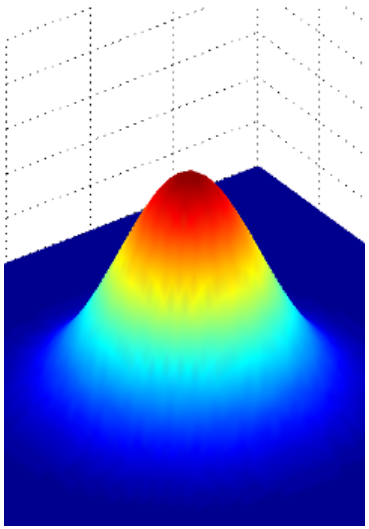
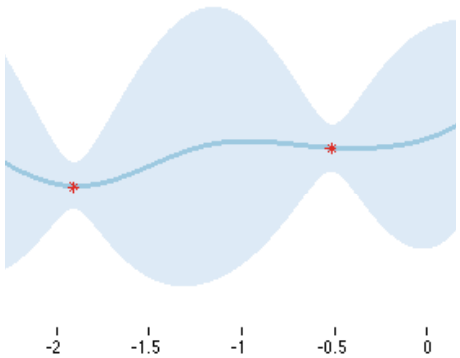
We define the generative model as





$$y_{nd} \overset{\text{iid}}{\sim} \text{Softmax}(f_{nd1}, \ldots, f_{ndK})$$

test result

$$\mathbf{y}_n = (y_{n1}, \ldots, y_{nD})$$

medical assessment

Continuous latent space of patients

Dist. over functions generating weights f_{ndk}

N patients $\mathbf{x}_n$, D categorical tests, each with K possible test results

Categorical Latent Gaussian Process

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1. **Sparse GPs** $\rightarrow$ linear time complexity.
2. **Monte Carlo integration** for the non-conjugate likelihood $\rightarrow$ noisy gradients.
3. **Learning-rate free stochastic optimisation** $\rightarrow$ no fine-tuning of learning-rates.
4. **Symbolic differentiation** $\rightarrow$ simple and modular code

```
import theano.tensor as T
X = m + s * randn(N, Q)
U = mu + L.dot(randn(M, K))
Kmm, Kmn, Knn = RBF(sf2, l, Z), RBF(sf2, l, Z, X), RBFnn(sf2, l, X)
KmmInv = sT.matrix_inverse(Kmm)
A = KmmInv.dot(Kmn)
B = Knn - T.sum(Kmn * KmmInv.dot(Kmn), 0)
F = A.T.dot(U) + B[:,None]**0.5 * randn(N, K)
S = T.nnet.softmax(F)
KL_U, KL_X = get_KL_U(), get_KL_X()
LS = T.sum(T.log(T.sum(Y * S, 1))) - KL_U - KL_X
LS_func = theano.function(['''inputs'''], LS)
dLS_dm = theano.function(['''inputs'''], T.grad(LS, m)) # and others
# ... and run RMS-PROP
```

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Back to Breast Cancer...

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Uniform	(Discrete) Bigram Dirichlet–Multinomial	(Linear) Latent Gaussian Model	**Categorical Latent Gaussian Process**
8.68	3.41	3.57 ± 0.208	$\mathbf{2.86 \pm 0.119}$

Model perplexity, predicting randomly missing test results

(Lots more results in the poster!)

Thank you

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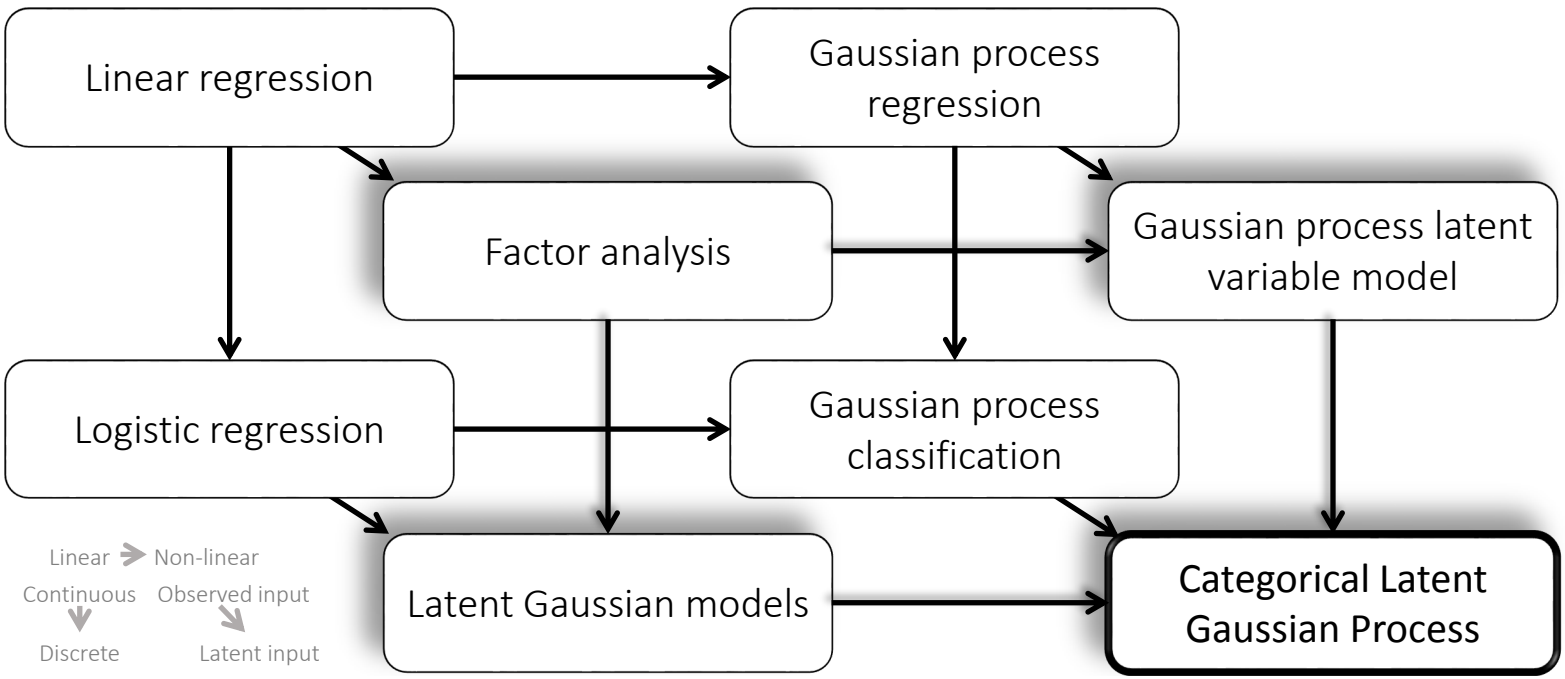

Please come and see our poster